Convection Parameterization in Atmospheric Modelling: Formulational Introduction

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Basic Reference:
Structure of Mass-Flux Convection Paprameterization:

Outline (Ingredients):

• Introduction: why we need this?
• Riehl & Malkus’ Hot Tower Hypothesis
• SCA
• entrainment-detainment
• environment
• Asymptotic limit: $\sigma_c \rightarrow 0$
• Closure
What should be parameterized: A Typical Atmospheric Convective System?

- Stratiform clouds (mesoscale)
- Cumulus convection (convective towers)

~200 km
~20 km
Parameterization Problem:

Full System (CRM) ? Parameterization (caricature)
**Q:** the subgrid-scale physical representations (parameterizations)?

**full physical system:** \[ \frac{\partial}{\partial t} \varphi = -\frac{1}{\rho} \nabla \cdot \rho \mathbf{v} \varphi + F, \]

average over the grid-box

\[ \frac{\partial}{\partial t} \bar{\varphi} + \frac{1}{\rho} \nabla \cdot \rho \bar{\mathbf{v}} \bar{\varphi} = -\frac{1}{\rho} \nabla \cdot \rho \mathbf{v}' \varphi' + \bar{F} \]

parameterization?

\[ \varphi' = \varphi - \bar{\varphi} = \text{(Downscaling)} \]
Examples:

Heat Equation:
Temporal Tendency + ‘Large-Scale’ Advection
= Diabatic Heating + Subgrid-Scale Transport
= \( Q_1 \): Apparent Heat Source?

Moisture Equation:
Temporal Tendency + ‘Large-Scale’ Advection
= -Condensation + Evaporation + Subgrid-Scale Transport
= -\( Q_2 \): Apparent Moisture Sink?
Observational Estimates of Q1 and Q2: Deep convection

Tropical Pacific

Tropical Atlantic

Note the typical tropical maximum of Q1 at 500 hPa, Q2 maximum is lower and typically at 800 hPa

Yanai et al., 1973, JAS

Yanai and Johnson, 1993
Why It Is Important?

Because of Its Global Role:

Especially over the Tropics (Riehl and Malkus 1958):

Hot-Tower Hypothesis (for the Hadley Circulation)
Hot-Tower Hypothesis
(Riehl and Malkus 1958):

Classcial View on Hadley Circulation:

Modern View:

Hot Towers (~5km)
Hot-Tower Hypothesis
(Riehl and Malkus 1958):

Large-Scale Upward Motion

Radiative Cooling

Dry entropy

Entropy Transfer

Moist entropy
Hot-Tower Hypothesis
(Riehl and Malkus 1958):

Radiative Cooling

Hot Tower

Dry entropy

Moist entropy
Hot-Tower Hypothesis
(Riehl and Malkus 1958): Implications:

Hot Towers $\ll$ Large-Scale Circulation
(Hadley-Walker)

Parameterization
(Convective Parameterization Problem)

cf., Scale-Separation Principle
Hot towers within a grid box:
Basic idea of convection parametrization
Hot towers within a Gird Box: Basic idea of Convection Parametrization

(Fig. 1, Arakawa and Schubert 1974)
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Why It Is Important?: Historical Perspectives

• Role in Tropical Large-Scale Circulation (Riehl and Markus 1958):
  Convective Parameterization (Arakawa & Schubert 1974, Kuo 1974, etc)

• Weather Modification Programme (1960th):
  Verification? → One-Dimensional Plume Model

• Natural Laboratory ↔ Fluid Laboratory Experiments (ca., 1950-1965):
Historical Evolution of the Study

Laboratory Experiments (50-60)

1D Entraining-Plume Model (EPM)
- (steady, 60th)
- (parameterization)

(explicit studies)

Time-dependent 1D EPM (Asai & Kasahara 1967)

+ Mircophysics (Ogura & Takahashi 1971)

2D CRM (70-80)

3D CRM (80-90)

Comparisons

Cloud Resolving Model (CRM)

MassFlux Convective Parameterization

?
Basic Questions

Physics:
- Conditional Instability, CAPE
- Convective-Radiative Equilibrium
- Quasi-Equilibrium
- Convective Parameterization (MassFlux):
  - Convective-Radiative Equilibrium
  - Quasi-Equilibrium
  - Q1, Q2, MassFlux
- Hot Tower: Plume Model?:
  - Entrainment-Detrainment
Basic Physics: Complex:

Dynamics: Buoyancy-Driven

Thermodynamics: Transport=Diffusion +Latent Heating

Cloud Microphysics: Rain Formation

Chemistry (Aerosoles): CCN

Radiation: Solar and Infrared

Electricity: Lightning: Electro-Magnetic
Hot-Tower Hypothesis (Riehl and Malkus 1958): Implications:

**Hot Towers** $\ll$ **Large-Scale Circulation** (Hadley-Walker)

Parameterization (Convective Parameterization Problem)

cf., Scale-Separation Principle
Hot towers within a Gird Box: Basic idea of Convection Parametrization

Grid Box = Convection + Environment
Geometrical Constraint (on CRM): **SCA** (Segmentally-Constant Constant Approximation)

**Grid Box** = Convection + Environment

- Hot Towers (Convection): homogeneous inside
- Environment: homogeneous
SCA: Segmentally-Constant Approximation: Side View

$w_e, \theta_e, \ldots, \varphi_e$

$x$

Convective Updraft (Hot Tower)

$w_c, \theta_c, \ldots, \varphi_c$

Environment
Segmentally-Constant Approximation (SCA) Into Cloud-Resolving Model (CRM) or Nonhydrostatic Anelastic Model (NAM)?

: NAM-SCA
A Hot Tower (Plume): Segmentally-Constant Approximation (SCA) Into CRM (NAM)?

Finite Volume Method
2D NAM-SCA Basic Formulation

The full system:

$$\frac{\partial}{\partial t} \varphi + \frac{\partial}{\partial x} u \varphi + \frac{1}{\rho} \frac{\partial}{\partial z} \rho w \varphi = F$$

Apply SCA over a segment \([x_j, b, x_{j+1}, b]\):

$$\int_{x_1}^{x_2} \left[ \frac{\partial}{\partial t} \varphi + \frac{\partial}{\partial x} u \varphi + \frac{1}{\rho} \frac{\partial}{\partial z} \rho w \varphi \right] dx = \int_{x_1}^{x_2} F \, dx$$

or

$$\frac{\partial}{\partial t} \sigma_j \varphi_j + \frac{1}{L} [(u_{j+1, b} - \dot{x}_{j+1, b}) \varphi_{j+1, b} - (u_{j, b} - \dot{x}_{j, b}) \varphi_{j, b}] + \frac{1}{\rho} \frac{\partial}{\partial z} \rho \sigma_j (w \varphi)_j = \sigma_j F_j$$

Mass continuity:

$$(u_{j+1, b} - u_{j, b}) + \frac{1}{\rho} \frac{\partial}{\partial z} \rho L_j w_j = 0$$

NAM: Nonhydrostatic Anelastic Model
**Hierarchy (3):** SCA: segmentally–constant approximation

\[
\frac{\partial}{\partial t} \sigma_j \varphi_j + \frac{1}{\rho} \frac{\partial}{\partial z} \rho \sigma_j (w \varphi)_j + \frac{1}{S} \int_{\partial S_j} \varphi (v^*_H - \dot{r}_b) \cdot d\mathbf{r} = \sigma_j F_j \tag{3.1}
\]

where

\[
\varphi_j = \frac{1}{S_j} \int_{S_j} \varphi dxdy, \tag{3.2a}
\]

\[
v^*_H = v_H - w \frac{\partial r_{j,b}}{\partial z}. \tag{3.2b}
\]

\(r_{j,b}\): the segment boundary position

\(\sigma_j = S_j / S\): the fractional area occupied by the \(j\)-th segment

**NB:** two major approaches are possible:

i) bulk: a single plume + environment : \(N = 2\)

ii) spectrum: \(N > 2\)
A Hot Tower (Plume):
Segmentally-Constant Approximation (SCA)
Into CRM (NAM)
Updraft + Downdraft: A simple demonstration Without environment Hypothesis with NAM-SCA:
Truncated NAM-SCA:
More Physical Processes:
Truncated NAM-SCA:
Final SCA-Archtype Model?

2D 4-segment model:

Axisymmetric 3-segment model:
Numerical Archetype: Fixed Distribution of Finite-Volume Elements
Tropical Squall-line (GATE Phase III)
Generalization of Concept:

Hot Towers < Plumes < Convection
NAM-SCA: Basic Formulation:

\[
\frac{d\sigma_j w_j}{dt} = \ldots
\]

\[
\frac{d\sigma_j \theta_j}{dt} = \ldots
\]

\[
\frac{d\sigma_j \varphi_j}{dt} = \ldots
\]

\[u_j = \text{mass continuity} \quad \leftarrow \ w_j\]
Simplification of the Horizontal Velocity Calculation:

Entrainment-Detainment Hypothesis
Mass Continuity:
Vertical Velocity $\leftrightarrow$ Divergence

NAM-SCA:

Entrainment-Detrainment hypothesis:
Prescribed Entrainment-Detrainment $\rightarrow$ Vertical Velocity (Mass Flux)
Entrainment-Detrainment hypothesis:

Prescribed Entrainment-Detrainment → Vertical Velocity (Mass Flux)

\[
\frac{1}{S} \int_{\partial S_j} \rho \mathbf{v}_H \cdot d\mathbf{r} = D_j - \sum_{i \neq j} E_{j,i} = -\frac{1}{\rho} \frac{\partial}{\partial z} \rho \sigma_j \omega_j
\]

\[
\frac{1}{S} \int_{\partial S_j} \rho \varphi \mathbf{v}_H \cdot d\mathbf{r} = D_j \varphi_j - \sum_{i \neq j} E_{j,i} \varphi_i
\]

Mass flux
Historical Note:
Extensive Laboratory Experiments during 1940s-1950s in order to understand Atmospheric Convection
Entraining Plume Hypothesis:
Laboratory Experiments
(Morton et al., 1956)
Entraining Plume Model: Laboratory Experiments

(Morton et al. 1956) (Turner 1962)
Entraining–Plume Hypothesis

\[ \frac{dM}{dz} = \epsilon M \]

where

\[ \epsilon = \text{constant: (fractional) entrainment rate} \]
\[ M = \rho \sigma w : \text{(convective) mass flux} \]
Further Historical Notes:

• **Role in Tropical Large-Scale Circulation** (Riehl and Markus 1958):

• **Natural Laboratory ↔ Fluid Laboratory Experiments (ca., 1950-1965, Morton, et al)**

• **Weather Modification Programme (1960th, Markus-Simpson): Vertification?**

• **One-Dimensional Plume Model**

• **Convective Parameterization** (Arakawa & Schubert 1974, Kuo 1974, etc)
Entrainment–Detrainment Hypothesis

\[ \frac{dM}{dz} = (\epsilon - \delta)M \]

\[ \frac{dM \varphi_c}{dz} = (\epsilon \varphi_e - \delta \varphi_c)M \]

where

\( \epsilon \): (fractional) entrainment rate
\( \delta \): (fractional) detrainment rate
\( M = \rho \sigma w \): (convective) mass flux
\( \varphi_c \): convective component
\( \varphi_e \): environment
Various Possibilities for Entrainment-Detrainment Hypotheses (cf., de Rooy et al, 2013, QJ):

after Raymond, 1993

- Undiluted entraining plume
- Cloud top entrainment
- Stochastic mixing

Hot tower

Stommel (1951)

Paluch (1979)

Raymond & Blyth (1986)
Environement-Convection Separation (a hidden assumption)

NB: Additional Constraint that can be removed
Environnement-Convection separation

convective element (a) Environment convective element (b)
Hierarchy (5): Environment Hypothesis:

\[ E_{j,i} = E_j \delta_{i,e} \]  \hspace{1cm} (5.1)

then

\[ \frac{1}{S} \oint_{\partial S_j} \varphi(\mathbf{v}_H^* - \mathbf{r}_{j,b}) \cdot d\mathbf{r} = D_j \varphi_j - E_j \varphi_e \]  \hspace{1cm} (5.2a)

&

\[ \frac{\partial}{\partial t} \sigma_j \varphi_j + \frac{1}{\rho} (D_j \varphi_j - E_j \varphi_e) + \frac{1}{\rho} \frac{\partial}{\partial z} \rho \sigma_j w_j \varphi_j = \sigma_j F_j \]  \hspace{1cm} (5.2b)

\[ \rho \frac{\partial}{\partial t} \sigma_j + \frac{\partial}{\partial z} \rho \sigma_j w_j = E_j - D_j \]  \hspace{1cm} (5.2c)

here \( \epsilon_j = 0 \), for plume elements, and

\[ \frac{1}{S} \oint_{\partial S_e} \varphi(\mathbf{v}_H^* - \mathbf{r}_{j,b}) \cdot d\mathbf{r} = \sum_j (E_j \varphi_e - D_j \varphi_j) + \nabla \cdot \mathbf{v}_H \varphi_e \]  \hspace{1cm} (5.3a)

\[ \frac{\partial}{\partial t} \sigma_e \varphi_e + \frac{1}{\rho} (E \varphi_e - \sum_j D_j \varphi_j) + \frac{1}{\rho} \frac{\partial}{\partial z} \rho \sigma_e w_e \varphi_e + \nabla \cdot \mathbf{v}_H \varphi_e = \sigma_e F_e \]  \hspace{1cm} (5.3b)

for the environment
MassFlux Parameterization

Last Steps:

• Fractaional area : $\sigma_c \rightarrow 0$

• Plumes are at steady state: $d/dt = 0$

\[
\frac{1}{\rho}(D_j q_j - E_j q_e) + \frac{1}{\rho} \frac{\partial}{\partial z} M_j q_j = \sigma_j F_j
\]

\[
\frac{\partial}{\partial z} M_j = E_j - D_j
\]

NB: the Formulation is time-dependent Before taking this Limit
Standard Mass-Flux Formulation:

\[ M = \eta(z)M_B(t) \]

\( \eta(z) \): cloud model: steady (no trigger, etc)

\( M_B(t) \): closure condition: ?

large-scale control

NB: if a life-cycle (e.g., trigger) of individual convection is to be considered, the above formulation is no longer valid (cf., NAM-SCA formulation)
Closure Problem (cf., Yano et al., 2013, ACP):

Two Major Possibilities based on conservation laws:

• Moisture (Kuo 1974)
• Energy Cycle: CAPE, Cloud Work Function (Arakawa and Schubert, 1974, Yano and Plant 2012a, b, Plant and Yano 2013)

• NB: Trigger and Suppression are NOT a Formal Part of Closure (cf., Yano et al., 2013, ACP)
Structure of Mass-Flux Convection Parameterization:

Ingredients (Summary):

- **Hot Towers**: SCA \( \text{NAM-SCA} \)
- **entrainment-detainment**
- **environment**
- **Asymptotic limit**: \( \sigma_c \rightarrow 0 \)
- **Closure**
- **Prognostic**:
  - High-Resolution Limit?
  - Quasi-Equilibrium (Yano and Plant, 2013, RG)
Further Discussions?:
Removal of the Environment Hypothesis

: Truncated NAM-SCA
When the Entrainment-Detrainment Hypothesis is reintroduced:
SCA-Plume Experiments (NAM-SCA, Yano and Baizig 2012)

Fixed interface:

Lagrangian:
Fractional Entrainment-Detrainment Rate

Fixed interface

(a) $\mu = 1$
(b) $\mu = 0.8$
(c) $\mu = 0.6$
(d) $\mu = 0.4$
(e) $\mu = 0.2$
(f) $\mu = 0.1$
(g) $\mu = 0$
(h) $\mu = -0.1$
(i) $\mu = -0.3$

Lagrangian $K$

0 4 8 12 16 20

0 4 8 12 16 20

$\mu = 1$
$\mu = 0.8$
$\mu = 0.6$
$\mu = 0.4$
$\mu = 0.2$
$\mu = 0.1$
$\mu = 0$
$\mu = -0.1$
$\mu = -0.3$
Dependence on Aspect Ratio:

Fractional Entrainment-Detrainment Rate
Experiment with Entraining-Plume Hypothesis:
In General:

\[
\frac{\partial}{\partial t} \sigma_j u_j + \frac{\partial}{\partial p} \sigma_j \omega_j u_j + d_j u_j - \sum_{i=\{i\}} e_{j,i} u_i + \nabla \cdot (\sigma_j u_j u_j) = -\nabla \sigma_j \phi_j + \sigma_j F_{u,j}.
\]

\[
\frac{\partial}{\partial p} (\sigma_j \phi_j) = -\sigma_j \alpha_j.
\]

\[
\frac{\partial \sigma_j \omega_j}{\partial p} = -\nabla \cdot (\sigma_j v_j)
\]

\[
\frac{\partial}{\partial t} \sigma_j \varphi_j + \frac{\partial}{\partial p} \sigma_j (\omega \varphi)_j + d_j \varphi_j - \sum_{i=\{i\}} e_{j,i} \varphi_i + \nabla \cdot (\sigma_j u_j \varphi_j) = \sigma_j F_j
\]

\[
\frac{\partial}{\partial t} \sigma_j = e_j - d_j.
\]
Robust Basis for Parameterization Closure: Convective Energy Cycle (Yano and Plant 2012a, b, QJ, JAS, Plant and Yano 2013 DAO)
Closure Problem in Mass-Flux Formulation (Arakawa and Schubert 1974):

Convective Energy-Cycle System:

Kinetic Energy Equation (Eq. 132, AS74)
Cloud-Work Function Equation (Eq. 142, AS74)

Three unknowns: \( K, M, A \)

A Functional Constraint: \( K \sim M^p \)

\( p=2: \) Pan and Randall, \( p=1: \) Yano and Plant

Prognostic Closure based on Energy Cycle

NB: Key is to couple the shallow and deep modes under this Formulation in order to properly describe Shallow-to-Deep Transformation (Yano and Plant, accepted to JAS)
Furhter Extension of NAM-SCA: Put More Plumes

+ Time-Dependent Activation-Deactivation of Plumes (Segments)

\[ \downarrow \]

Highly-Flexible Adaptive Mesh-Refinement: NAM-SCA
Activation:

interface jump $\rightarrow$ fac $\times$ variance (z)
Deactivation:

interface jump < facd \times \text{variance (z)}

\[ \text{fac} = \text{facd} = 1 \]
Dry Convective Boundary Layer
Dry Convective Boundary Layer

time = 8\tau

\begin{align*}
&\begin{array}{c}
\text{W} \\
(\text{m/s})
\end{array} \\
\begin{array}{c}
2.00 \\
-1.00
\end{array} \\
\text{m/s}
\end{align*}

\begin{align*}
&\begin{array}{c}
\theta \\
(\text{K})
\end{array} \\
\begin{array}{c}
1.00 \\
-1.00
\end{array} \\
\text{K}
\end{align*}

time = 9\tau

\begin{align*}
&\begin{array}{c}
\text{W} \\
(\text{m/s})
\end{array} \\
\begin{array}{c}
2.00 \\
-1.00
\end{array} \\
\text{m/s}
\end{align*}

\begin{align*}
&\begin{array}{c}
\theta \\
(\text{K})
\end{array} \\
\begin{array}{c}
1.00 \\
-1.00
\end{array} \\
\text{K}
\end{align*}
More Applications:

Archetype Convection Representation
Application I: Moncrieff’s Archetype Convection Representation (1992)?

NAM-SCA Framework can numerically generate an archetype for a given environmental state (Cubism Version)

Figure 1. (a) Schematic diagram of the relative airflow and physical processes associated with a small line MCS. (From Rutledge 1991, adapted from Houze et al. 1989) (b) Schematic diagram of the airflow in the stationary dynamical model showing the three flow branches, namely jump updraught (A); downdraught (B); and overturning updraught (C). Branches have piecewise constant densities $\rho_n \leq \rho_\Lambda \leq \rho_i$ with $\rho_n = \rho_{\Lambda} = \rho_i$ in the archetypal model.
Cloud Water (g/kg)

Original NAM-SCA Run
Cloud Water (g/kg)

NAM-SCA Run in Moving Coordinate: Initial Condition For Archetype
Finite-Volume Element Distribution (original)

NAM-SCA Run in Moving Coordinate: Initial Condition For Archetype
Numerical Archetype: Fixed Distribution of Finite-Volume Elements (smoothed)
Cloud Water (g/kg)

NAM-SCA Run in Moving Coordinate: Initial Condition For Archetype
Numerical Archetype: Fixed Distribution of Finite-Volume Elements
Cloud Water (g/kg)

Numerical Archetype: Fixed
Distribution of Finite-Volume Elements